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A relation between the charge radii of π^+, K^+, K^0 derived by the general QCD parametrization

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Abstract. We derive, using the general QCD parametrization, the approximate relation $r^2(\pi^+) - r^2(K^+) \cong -r^2(K^0)$ where the r's are the charge radii. The relation is satisfied but the experimental errors are still sizeable. The derivation is similar to (but even simpler than) that for $r^2(p) - r^2(n) \cong r^2(\Delta^+)$ (Phys. Lett. B 448, 107 (1999)).
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1. Introduction

It has been shown in a series of papers [1, 2] that one can parametrize exactly several hadron properties using only general features of QCD. The method, named the "general parametrization" (GP), allows to write, almost at first sight, the most general expression for the spin-flavor structure of quantities relevant to the lowest baryons (octet+decuplet) and mesons. It turns out that the coefficients of the various terms in the parametrization decrease with increasing complexity of the term; this is called the hierarchy of the parameters [1g,2a], and is the reason why the naive non relativistic quark model [3] (NRQM), that takes into account only the simplest terms, works reasonably well, in spite of the fact that the motion of the light quarks inside hadrons is relativistic. Indeed the GP method was conceived [1a] with the aim of explaining the fair success of the NRQM; it emerged that, besides

explaining this, the method leads to predictions in many cases, due precisely to the hierarchy. The GP, though non covariant, is relativistically correct, being an exact parametrization of a fully relativistic field theory. It is also independent [2a] of the choice of the QCD quark mass renormalization point.

In a previous paper [2f], we applied the GP to the electric charge radii of p, n and Δ^+ stimulated by a paper of Buchmann, Hernandez and Faessler [4] who derived the relation $r^2(p)-r^2(n) = r^2(\Delta^+)$ using a quark model including two body gluon and pion exchange. We showed, indeed, that such relation is obtained quite generally by the GP method, if one neglects three index terms and the cloosed loop contribution (absent in [4]); the expected order of magnitude of the modification due to the three index terms was found (10% to 20%)$\cdot r^2(\Delta^+)$.

Here we apply the GP method to the charge radii of π^+, K^+, K^0. The calculation is even simpler than that for p, n and Δ^+ because we are now dealing with mesons. Also, while the p, n, Δ^+ relation cannot be checked experimentally, so far, because the radius of the Δ^+ is not known, in the present case values for the meson radii, even if affected by large errors, exist [5, 6]. In $(fm)^2$ it is:

$$r^2(\pi^+) = 0.44\pm0.01 \quad ; \quad r^2(K^+) = 0.34\pm0.05 \quad ; \quad r^2(K^0) = -0.054\pm0.026 \quad (1)$$

2. The general parametrization of $r^2(\pi^+)$, $r^2(K^+)$, $r^2(K^0)$

The square radius $r^2(M)$ of a meson M with e.m. form factor $F(q^2)$ is:

$$r^2(M) = -6\frac{dF(q^2)}{dq^2}\Big|_{q^2=0} \quad ; \quad F(q^2) = \langle M(\boldsymbol{q}/2)|\rho(0)|M(-\boldsymbol{q}/2)\rangle \qquad (2)$$

Here $|M(\boldsymbol{p})\rangle$ is the exact eigenstate of the QCD Hamitonian for the meson M with total momentum $\boldsymbol{p}$, $\rho(0) = i\bar{\psi}(0)Q\gamma_4\psi(0)$, with $\psi(x)$ the quark field, and $Q = (1/2)(\lambda_3 + \frac{1}{3}\lambda_8)$ is the charge operator. Incidentally recall that the q^2 dependence of the e.m. form factors was discussed for p and n in ref.[2g].

Using the technique described in [1,2] (see [1a] or, more brifley, the Appendix of [2a,1i]), the exact $r^2(M)$ derived from QCD for a meson with an $L = 0$ auxiliary state (as the lowest pseudoscalar mesons) is written in the GP:

$$r^2(M) = \langle W_M | \text{“}parametrized\ r^2\text{”} | W_M \rangle \tag{3}$$

where W_M are the standard spin-flavor functions of the π, K mesons constructed in terms of the quark-antiquark spin flavor variables; W_M is the product of a singlet quark-antiquark spin factor times an octet flavor factor. The notation for mesons is similar to that in refs.[1c,1d], except that the projector Π^λ in [1d] is now called P^s (P^s is 1 acting on a strange q or $\bar{q}$; 0 otherwise).

In view of the linearity of r^2 in $\rho(0)$ the most general “*parametrized* r^2” for π^+, K^+, and K^0 is a scalar linear in the quark charges Q_i ($Q_i = Q_1, Q_2$; note: $1 = quark$, $2 = antiquark$). As to the the spins, the “*parametrized* r^2” can only contain $(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$, which, applied to the spin singlet factor in W_M, is just -3. Thus the most general “*parametrized* r^2” (a scalar under rotations, of course) is:

$$\text{“}parametrized\ r^2\text{”} = A \sum_i Q_i + B \sum_i Q_i P_i^s + C \sum_{i \neq k} Q_i P_k^s + DTr[QP^s] \tag{4}$$

In (4) A, B, C, D are four real parameters; the sums are on $i, k = 1, 2$; for instance, $\sum_{i \neq k} Q_i P_k^s = Q_1 P_2^s + Q_2 P_1^s$; for K^+, it is $\langle W_{K^+} | Q_1 P_2^s + Q_2 P_1^s | W_{K^+} \rangle = +(2/3)$.

Neglecting the $Trace$ term in (4) because, as usual [2a], it is associated to closed internal loops, depressed by the Furry theorem at least 50 times with respect to the dominant term A, the eq.(4) contains three parameters A, B, C and must fit three quantities. The hierarchy leads to $|B/A|$=the flavor reduction factor $\cong 1/3$ (more precisely 0.3 to 0.33). For $|C/B|$ (the reduction factor due to one more gluon exchange) we also take $|C/B| \approx 1/3$ (but here the 1/3 is less “universal”; that is, it depends in part on the quantity under consideration and may vary, say, between 0.2 and 0.37). In conclusion we set:

$$C \approx (1/3)B \cong (1/9)A \tag{5}$$

3. Discussion of the parametrized expression (4) for the radii

The eq.(4) gives (neglecting, as stated, the $Trace$ term):

$$\begin{array}{l} r^2(\pi^+) = A \quad (= 0.44 \pm 0.01) \\ r^2(K^+) = A + (1/3)B + (2/3)C \quad (= 0.34 \pm 0.05) \\ r^2(K^0) = (1/3)B - (1/3)C \quad (= -0.054 \pm 0.026) \end{array} \tag{6}$$

(Incidentally, omitting C, the eqs.(6) coincide with those of the NRQM in its most naive form. Note that, omitting C, we obtain from (6) $r^2(K^0) = -0.010 \pm 0.05$.) In general the eqs. (6) lead to the relation mentioned in the abstract:

$$\begin{array}{ccc} r^2(\pi^+) - r^2(K^+) & = & -r^2(K^0) + C \\ 0.10 \pm 0.05 & & 0.054 \mp 0.026 \pm 0.05 \end{array} \tag{7}$$

In the numbers below the relation (7), the last 0.05 is C, calculated using the hierarchy (5). One can see that the GP prediction (7), also here, agrees with the facts, although more precise measurements of $r^2(K^+)$ and $r^2(K^0)$ are needed. Incidentally, it is, from (6):

$$|B/A| = |r^2(K^+) - r^2(\pi^+) + 2r^2(K^0)|/r^2(\pi^+) = |0.10 \pm 0.05 + 0.108 \mp 0.052|/0.44 \tag{8}$$

As stated, the hierarchy leads us to expect unambigously $|B/A| \cong 1/3$. We note that, to obtain this value from eq.(8), it is necessary that the two errors in (8) combine to -0.06; the central values alone produce a too large $|B/A|$.

In conclusion two new results emerge: a) The predicted relation (7) between the r^2's of π^+, K^+, K^0 satisfied as indicated; b) The prediction (eq.(8)) that either $r^2(K^+)$ or $-|r^2(K^0)|$ (or both) must have their center values somewhat larger than the present ones.

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